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AUTHOR Ward, Barbara
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ABSTRACT

The National Assessment of Educational Progress (NAEP) has completed two surveys of mathematics achievement of 9-, 13-, and 17-year-old students. The first was conducted during the 1972-73 school year and the second five years later, during 1977-78. This report describes changes in students' performance between the assessments. The overall results indicate some decline in mathematics achievement. Changes in performance differed by type of item and by age group. Declines became more apparent for older students. The results for whole number computation were generally satisfactory; however, results for problem solving were generally low and had declined. The assessment indicated a significant decline on mathematics understanding items only for 17-year-olds, while all three age groups declined on mathematics applications items. A panel chosen to intepret the results warned against placing too much emphasis on overall results and stated that changes for population groups or specific item types provide more meaningful information. The panel made seven recommendations for school mathematics. (MK)

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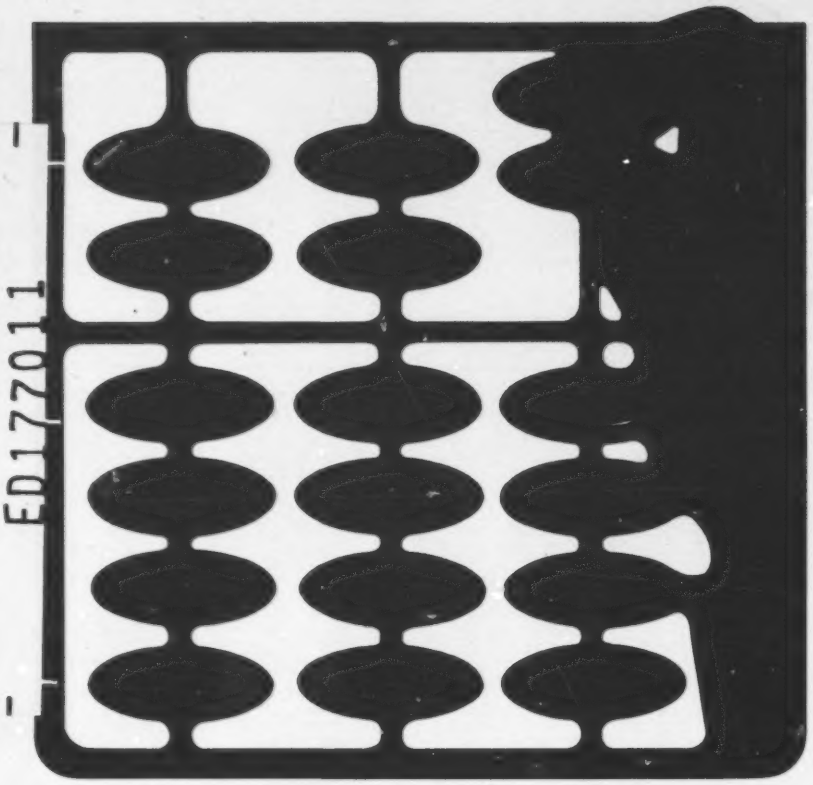

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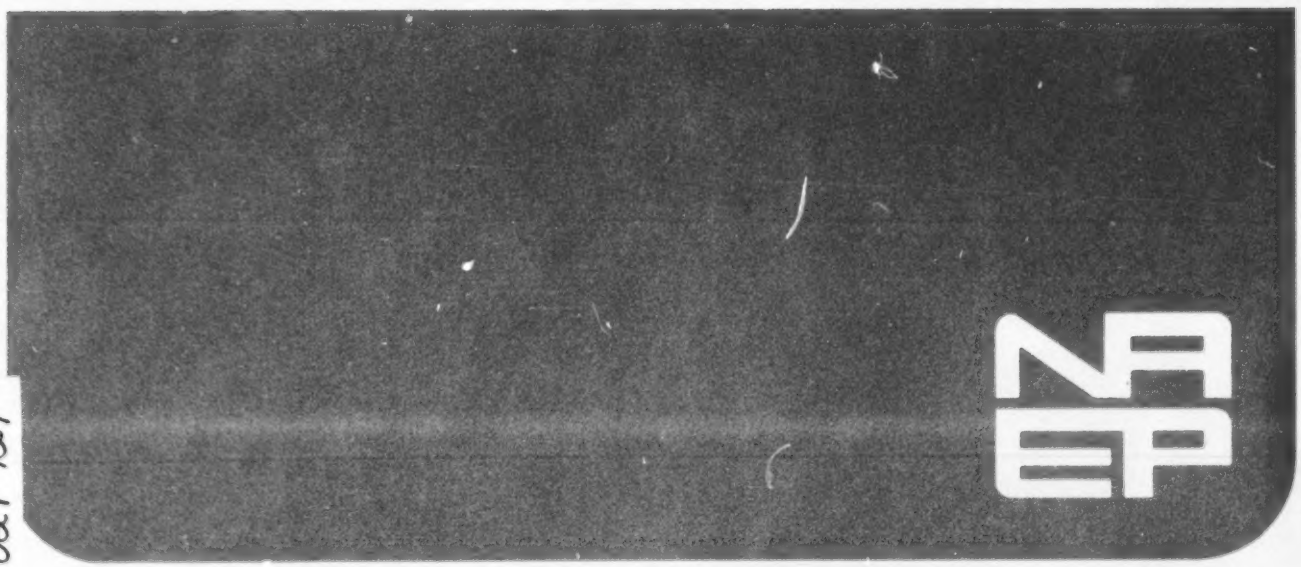

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CHANGES IN MATHEMATICAL ACHIEVEMENT, 1973 - 78

RESULTS FROM THE SECOND ASSESSMENT OF MATHEMATICS

Report No. 09-MA-01

by the
National Assessment of Educational Progress

Education Commission of the States
Suite 700, 1860 Lincoln Street
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The cost figure cited above represents the total amount of money expended since late 1973 on assessments in art, career and occupational development, reading, writing, social studies/citizenship, science, basic life skills, mathematics and consumerism, resulting, to date, in numerous reports, papers, articles, presentations and assessment materials, many of which are used in state and local assessment programs. A complete list of all such materials is available upon request.

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FOREWORD

When the U.S. Office of Education was chartered in 1867, one charge to its commissioners was to determine the nation's progress in education. The National Assessment of Educational Progress (NAEP) was initiated a century later to address, in a systematic way, that charge.

Each year since 1969, National Assessment has gathered information about levels of education achievement across the country and reported its findings to the nation. NAEP surveys the education attainments of 9-year-olds, 13-year-olds and 17-year-olds in 10 learning areas: art, career and occupational development, citizenship, literature, mathematics, music, reading, science, social studies and writing. Different learning areas are assessed every year, and all areas are periodically reassessed in order to measure possible changes in education achievement. National Assessment has interviewed and tested more than 810,000 young Americans since 1969.

Learning-area assessments evolve from a consensus process. Each assessment is the product of several years of work by a great many educators, scholars and lay persons from all over the nation. Initially, these people design objectives for each subject area, proposing general goals they feel Americans should be achieving in the course of their education. After careful reviews, these objectives are given to exercise (item) writers, whose task it is to create measurement instruments appropriate to the objectives.

When the exercises have passed extensive reviews by subject-matter specialists, measurement experts and lay persons, they are administered to probability samples. The people who compose these samples are chosen in such a way that the results of their assessment can be generalized to an entire national population. That is, on the basis of the performance of about 2,500 9-year-olds on a given exercise, we can make generalizations about the probable performance of all 9-year-olds in the nation.

After assessment data have been collected, scored and analyzed, National Assessment publishes reports to disseminate the results as widely as possible. Not all exercises are released for publication. Because NAEP will readminister some of the same exercises in the future to determine whether the performance level of Americans has increased, remained stable or decreased, it is essential that they not be released in order to preserve the integrity of the study.

ACKNOWLEDGMENTS

Many organizations and individuals have made substantial contributions to this report. Not the least of those to be gratefully acknowledged are the administrators, teachers and students who cooperated so generously during the collection of the data.

Special acknowledgment must go to Jane Armstrong of the National Assessment staff for coordination of development efforts and to Stuart Kahl and George Moreno for their assistance. Also to be acknowledged are the substantial contributions in guiding item development and report planning made by the members of the Mathematics Advisory Board: Dr. Sarah Herriot, Dr. Prince Jackson, Dr. Jeremy Kilpatrick, Dr. Donald Kreider, Dr. John LeBlanc and Dr. James Wilson.

Administration of the mathematics assessment was conducted by the Research Triangle Institute, Raleigh, North Carolina. Scoring and processing were carried out by the Measurement Research Center, Iowa City, Iowa, and by the National Assessment staff.

Mrs. Iris Carl, Dr. Thomas Carpenter, Dr. Robert Dilworth, Mr. Edgar Edwards Jr., and Dr. James Wilson contributed their time and knowledge interpreting the implications of the data. Their efforts helped greatly in putting the data into perspective.

The actual preparation of this report was a collaborative effort of the National Assessment staff. Special thanks must go to the following people: Ingrid Larsson and Charlotte Overbury for data processing support; Betty Homler for secretarial support; Ava Powell, Marci Reser and Paula Pitchford for production. Statistical analyses for this report were supervised by Donald Phillips. The report was written by Barbara Ward.

Roy H. Forbes
Director

INTRODUCTION

Teaching of mathematics has often been a center of controversy in recent years. Introduction of the "new math" in the 1960s and the "back to the basics" movement in the late 1970s have ignited considerable public debate, leaving many people uncertain about what the schools are or should be teaching in mathematics and apprehensive about what has happened to students in the meantime.

The National Assessment of Educational Progress (NAEP) has presently completed two surveys of the mathematics achievement of 9-, 13- and 17-year-old students, the first conducted during the 1972—73 school year and the second five years later, during 1977—78. This report describes changes in students' performance between the 1973 and 1978 assessments.

The items used to measure changes in achievement did not provide as broad of a coverage of the content of school mathematics as the entire 1977—78 mathematics assessment, and results should thus be interpreted with caution. For a complete picture of students' achievement in mathematics, change results should be considered together with the baseline results for the 1977—78 assessment, which focus upon mathematical knowledge and skills, mathematical understanding and mathematical applications (*Mathematical Knowledge and Skills*, 1979; *Mathematical Applications*, 1979; *Mathematical Understanding*, forthcoming).

The Mathematics Assessments

The 1977—78 mathematics assessment was designed to measure students' abilities at four different cognitive process levels crossing a variety of traditional mathematics content areas. These cognitive process levels are: (1) knowledge, (2) skills, (3) understanding and (4) application. Major content areas assessed are numbers and numeration; variables and relationships; geometry (size, shape and position); measurement; and "other topics," including probability and statistics, and graphs and tables. For a more complete description of the development of the 1977—78 mathematics assessment, refer to *Mathematics Objectives, Second Assessment* (1978).

The knowledge cognitive process level involves recall of facts and definitions, and includes such topics as number order; place value; basic facts of addition, subtraction, multiplication and division; geometric figures; and measurement units. Skills involve the ability to use specific algorithms and manipulate mathematical symbols. Included in the skills cognitive process level are computing with whole numbers, fractions, decimals, percents and integers; taking measurements; converting measurement units; reading graphs and tables; and manipulating geometric figures and algebraic expressions. Understanding implies a higher level of cognitive process than simply recalling facts or using algorithms. Exercises assessing mathematical understanding asked students to provide an explanation or illustration of different mathematical knowledges or skills, requiring a transformation of knowledge but not the application of that knowledge to solve a problem. Application requires the use of mathematical knowledge, skills and/or understandings — typically in problem-solving activities.

Measuring Changes in Achievement

Testing conditions must be as nearly the same as possible in each assessment to assure

an accurate measure of *changes* in performance. National Assessment makes every effort to hold conditions constant by reading instructions and items on tape to students and by using trained administrators, rather than classroom personnel, to conduct the assessment. Items used to measure change are identical in wording and format in each survey, and time allowed for students to respond is the same. Comparable samples of young people are drawn for each assessment year (Chromy et al., 1974).

As previously stated, identical sets of items are used in each assessment year to measure changes in performance. Overall changes in mathematics achievement are based on 55 item parts for 9-year-olds, 77 item parts for 13-year-olds and 102 item parts for 17-year-olds. Between 2,100 and 2,500 students at an age responded to an item in each assessment. Items were packaged in booklets, and any one student completed only one booklet of about 45 minutes in length. In 1977—78 there were 7 booklets for 9-year-olds, 11 for 13-year-olds and 12 for 17-year-olds. Approximately 17,000 9-year-olds, 27,000 13-year-olds and 27,000 17-year-olds participated in the 1977—78 mathematics assessment.

Thirteen-year-olds were assessed in October through December in both assessment years; 9-year-olds, in January and February; and 17-year-olds, in March and April. Thus, the amount of school experience for each age group was approximately the same in each assessment.

Item scoring also must remain consistent across assessment years. Approximately 20% of the items used to measure change were multiple-choice. These were scored by an optical scanning machine; the same answers were scored correct in each assessment year. The remaining 80% of the items were open-ended, meaning that respondents had to supply the correct answer. Scoring guides for open-ended items, which define categories of acceptable and unacceptable responses, were revised in 1978. All 1973 responses were rescored at the same time that the 1978 responses were scored, using the 1978 scoring guides, to insure that scoring of the two sets of data was the same.

Reporting the Data

Differences in average or item-level performance between assessments are described as changes *only* if the differences are statistically significant at the .05 level; differences that are not significant at that level are described as being "not appreciably different." However, it should be remembered that changes may be statistically significant but still not large enough to be meaningful. A positive difference in performance for a certain average or item indicates that more people made a correct response in 1978 than in 1973; a negative difference shows that fewer people made a correct response in the second assessment than the first.

Results for specific items as well as changes in average performance should be used in evaluating performance. While averages are useful for summarizing performance, they may be unduly affected by atypical results.

Average performance should not be used as an indicator of ability levels because averages are affected by the difficulty of the items assessed. Average performance levels cannot be compared across age levels or cognitive process levels, since averages for these were computed using different sets of exercises, which were not statistically equivalent. Differences in average performance between assessments are meaningful since identical sets of items were used in computing particular averages for each assessment.

Additional Information on the Mathematics Assessment

This report describes changes in mathematical achievement between 1973 and 1978. The following three reports describe performance on the 1977—78 mathematics assessment

and are available from National Assessment: *Mathematical Knowledge and Skills* (1979), *Mathematical Understanding* (forthcoming) and *Mathematical Applications* (1979).

Technical documentation of the 1977—78 assessment includes a released exercise set showing items released for publication, with national and selected group results; a microfiched data appendix, providing additional group results for released exercises; and a yearbook describing assessment procedures. Reports and data volumes describing detailed results from the 1972—73 mathematics assessment are also available.

CHAPTER 1

CHANGES IN MATHEMATICS ACHIEVEMENT

The two NAEP mathematics assessments provide data on changes in mathematics achievement between the early and late 1970s. During the early 1970s, public concern over declining test scores and what was perceived as experimentation in the schools — for example, open classrooms, ungraded schools, performance contracting — led to the demand that education go "back to the basics." During the mid-1970s, mathematics textbooks emphasizing the basics of computational skills and knowledge of facts and definitions came into more prevalent use in the schools. In the same time period, schools were increasingly expected to be "accountable" for students' performance, and minimal competency requirements were established in a number of areas. Some commentators have also discerned changes in the society between the early and late 1970s — from the intense public political concern of the Viet Nam and Watergate eras to a preoccupation with more personal ambitions and goals.

TABLE 1. Changes in Average Performance on All Mathematics Items Assessed in 1973 and 1978, Ages 9, 13 and 17

	Number of Items	Average Performance 1973	Average Performance 1978	Change in Average Performance
Age 9	55	38%	37%	-1
Age 13	77	53	51	-2*
Age 17	102	52	48	-4*

**Change is significant at the .05 level.*

What happened to mathematics achievement during that time? When all items were considered together, 9-year-olds' performance declined very slightly; the decline for 13-year-olds was slightly larger and the decline for 17-year-olds was appreciable (Table 1). The change for 9-year-olds was very close to being significant at the .05 level ($p < .06$).

It is both interesting and informative to compare the changes in mathematics achievement with the changes observed for other subject areas assessed by National Assessment. In reading, 9-year-olds displayed a slight improvement between 1971 and 1975, while achievement of 13- and 17-year-olds did not change. Quality of writing, as displayed in written paragraphs, stayed about the same for 9-year-olds between 1970 and 1974, but the quality and coherence of 13- and 17-year-olds' written expression declined. Basic mechanics of writing — spelling, punctuation and capitalization — showed few changes at any age level. Nine-year-olds' social studies achievement did not change from 1972 to 1976; some declines were evident for 13-year-olds and larger drops occurred for 17-year-olds. Both teenaged groups also declined in their knowledge and attitudes regarding politics. In science, the achievement level of all three ages decreased between 1970 and 1973; however, between 1973 and 1977, achievement of 17-year-olds continued to fall while that of both younger groups stayed about the same.

While patterns across subject areas are not completely consistent, some trends are apparent. There is a fairly strong pattern of declines for 17-year-olds, the only exception being in the area of reading. Results for 9-year-olds, however, usually did not change greatly between the early and mid-1970s, and

improved somewhat in reading. Thirteen-year-olds were between these extremes, showing declines in some areas and not in others. Generally, their declines were not quite as large as those seen for the older teenagers. Thus, changes seen in mathematics achievement are basically in accord with patterns found for other subject areas.

The following sections describe changes in performance for the different cognitive process levels — knowledge, skills, understanding and application. Changes both in average achievement levels and in performance on specific exercises are discussed.

Mathematical Knowledge

Items assessing knowledge stressed recall and recognition of facts and definitions — including the names of numbers, number order and the names of geometric figures. Average results for all items in this section showed no appreciable change for any of the three age groups (Table 2). It should be remembered that changes in average performance represent the difference between 1973 and 1978 average performance on a set of items. Changes on individual items may vary considerably from the average.

TABLE 2. Changes in Average Performance on Mathematics Knowledge Items Assessed in 1973 and 1978, Ages 9, 13 and 17

	Number of Items	Average Performance 1973	Average Performance 1978	Change in Average Performance
Age 9	17	55%	55%	-1†
Age 13	16	64	64	0
Age 17	18	63	63	0

†Figures do not total because of rounding.

Average results can be greatly affected by a small number of outlying exercises. Exercises assessing knowledge of the metric system of measurement — one administered to 13-year-olds and two administered to 17-year-olds — showed very substantial gains (26 percentage points for the 13-year-olds' item; 12 and 15 percentage points for the 17-year-olds' items), whereas results for all but one other item measuring knowledge either displayed no change or declined. When results for the metrics items were removed from the knowledge averages, both 13- and 17-year-olds experienced declines (Table 3).

TABLE 3. Changes in Average Performance on Mathematics Knowledge Items Assessed in 1973 and 1978, Excluding Knowledge of Metric Measures, Ages 13 and 17

	Number of Items	Average Performance 1973	Average Performance 1978	Change in Average Performance
Age 13	15	67%	65%	-2*
Age 17	16	63	62	-2*†

**Change is significant at the .05 level.*
†Figures do not total because of rounding.

The limited number of items in the knowledge section does not allow statements about changes in knowledge of any particular content area. Results on specific exercises are described to indicate both the type of items used to assess change and the type of data comprising the average results.

Nine-year-olds' performance did not change on two items requiring them to write and order whole numbers. Approximately 88% in each assessment correctly wrote "five hundred twenty-two" in numerical form after seeing and hearing it in the verbal form. About 85% correctly ordered 4 two-digit numbers, several of which contained the same digits in different places. Thirteen-year-olds were even more successful on this task — around 96% of them correctly ordered the four numbers in each assessment.

Several items requiring the ordering of fractions and decimals seemed more difficult for teenagers in 1978 than in the previous survey. In 1978, 52% of the 13-year-olds and 81% of the 17-year-olds correctly selected a common proper fraction falling between two

given common proper fractions. Performance on this item dropped from the 1973 assessment – 6 percentage points for 13-year-olds and 4 percentage points for 17-year-olds. Relatively few 13-year-olds in either assessment — about 15% — identified a decimal number falling between two given decimal numbers. Forty-six percent of the 17-year-olds were successful at this task in 1978, a decline of 7 percentage points from the previous assessment.

Nine-year-olds' performance also did not change on several items dealing with number theory and the properties of numbers. Around 70% in each assessment could list two even numbers and two odd numbers. About 60% knew that a number multiplied by one equals itself, and about half knew that a number divided by one equals itself.

Relatively few items measured ability to recognize geometric figures; changes on these items generally were slight. Recognition of names of solids increased considerably from ages 9 to 13 to 17; at all three ages, more students recognized a rectangular solid than a sphere or a cylinder.

Approximately 12% of the 17-year-olds gave the correct definition for each of three trigonometric functions of an angle — the sine, cosine and tangent; this percentage did not change between assessments.

Changes in knowledge of measurement may reflect a shift toward greater emphasis on the metric system in the schools' curricula. As seen in Exhibit 1, knowledge of English unit relationships declined for all three ages, although the overall decline was not as pronounced for the 17-year-olds. On several items dealing with the metric system of measurement, 13- and 17-year-olds showed substantial improvements between 1973 and 1978. Sixty-three percent of the 13-year-olds in 1978, compared with 37% in 1973, correctly identified a kilometer as the largest unit in a list of metric units, an improvement of 26 percentage points. Sixty-nine percent of the 17-year-olds completed this task correctly, an improvement of 15 percentage points from 1973. On another metric exercise, in which 17-year-olds were asked to name the English unit most nearly equivalent to a metric unit, performance improved by 12 percentage points, to 71% in the second assessment.

EXHIBIT 1. Percentages of Correct Responses on Measurement Items Assessed in 1973 and 1978, Ages 9, 13 and 17

	Age	Percent Correct 1973	1978	Change
A. Which is longer?				
2 feet	9	83	79	- 4*
1 yard**	13	94	93	- 1
I don't know.	17	97	96	- 1
B. Which is heavier?				
17 ounces**	9	36	29	- 7*
1 pound	13	72	63	-10*†
I don't know.	17	86	84	- 2
C. Which holds more water?				
3 pints	9	84	81	- 3
2 quarts**	13	92	91	- 1
I don't know.	17	95	93	- 2*

EXHIBIT 1 *(Continued).* **Percentages of Correct Responses on Measurement Items Assessed in 1973 and 1978, Ages 9, 13 and 17**

	Age	Percent Correct 1973	Percent Correct 1978	Change
D. Which is a longer period of time?				
1 month	9	54	51	- 3
5 weeks**	13	86	83	- 4*†
I don't know.	17	95	95	--
E. Which is worth more?				
2 dimes**	9	83	85	2
3 nickels	13	92	93	1
I don't know.	17	93	94	--†

**Change is significant at the .05 level.*
***Correct answer.*
†Figures do not total because of rounding.
--Denotes difference less than 1%.

Mathematical Skills

Mathematical skills involve the ability to manipulate mathematical symbols or use an algorithm — for example, adding a column of numbers, reading information from a table or solving a given equation. Average results for skill items showed that 9-year-olds' skill levels did not change between 1973 and 1978. However, average skill levels of 13- and 17-year-olds declined, with 17-year-olds showing the larger drop. Results for items measuring specific mathematical skills are described in Table 4.

TABLE 4. Changes in Average Performance on Mathematics Skills Items Assessed in 1973 and 1978, Ages 9, 13 and 17

	Number of Items	Average Performance 1973	Average Performance 1978	Change in Average Performance
Age 9	21	26%	26%	0
Age 13	37	51	49	-2*
Age 17	46	55	50	-5*

**Change is significant at the .05 level.*

Computation is often viewed as one of the most basic mathematical skills, and it is in this area that much of the attention of the "basics" movement in mathematics has focused. Computation involves the four operations — addition, subtraction, multiplication and division — with whole numbers, fractions, decimals, percents, integers and exponents.

Mathematics curricula are fairly uniform nationwide in the order and often even in the grade level in which various skills and concepts are introduced. We would expect teachers to introduce addition of whole numbers first, followed by subtraction, multiplication and division of whole numbers. Computation with other types of numbers would usually follow computation with whole numbers. Of course, students do not have to completely master an operation before another is introduced. For example, most 9-year-olds would have developed some skill in multiplication, even though they might not be able to add large columns of numbers successfully.

Current mathematics textbooks indicate that fourth graders (three-fourths of the 9-year-olds in the National Assessment sample) are expected to be familiar with addition and subtraction, with and without regrouping

(carrying and borrowing), and simple multiplication. They probably would have been introduced to the concept of division. Third graders (one-fourth of the 9-year-old sample) would generally have less experience, especially with multiplication and division. One would expect 13-year-olds (seventh and eighth graders) to be acquainted with operations involving fractions, decimals, percents and integers. These concepts are often introduced in the upper primary grades and repeated with increasing complexity in successive years.

Figure 1 shows skill levels in 1973 and 1978 on various whole number computational tasks. On the first item ("What is the sum of 21 and 54?"), which does not require regrouping (carrying), approximately three-quarters of the 9-year-olds and just over four-fifths of the 13-year-olds were successful in 1978. Nine-year-olds improved between 1973 and 1978 while 13-year-olds declined. Nine-, 13- and 17-year-olds' abilities to add the column of numbers with regrouping did not change appreciably over the five-year period. On the subtraction item, which required students to regroup (borrow) twice, performance of 9- and 13-year-olds did not change, but that of 17-year-olds dropped by 4 percentage points. Ability to solve the simple multiplication item remained constant for all three age groups; however, 17-year-olds declined in their ability to solve the more complex multiplication item. Skill in answering a simple long division exercise stayed about the same for both groups of teenagers, with 71% of the 13-year-olds and 85% of the 17-year-olds answering correctly in 1978.

On these examples, the whole number computational skills of 9- and 13-year-olds appear to have remained relatively stable. Seventeen-year-olds may have lost some ground in their skills with more complex computations.

Exhibit 2 shows two items concerning computation with fractions and percents. Results declined appreciably on both items.

EXHIBIT 2. Percentages of Correct Responses on Two Computation Skills Items Assessed in 1973 and 1978, Ages 9, 13 and 17

What does 2/3 of 9 equal?

Answer (6)

	Percent Correct 1973	1978	Change
Age 9	18	13	-5*
Age 13	56	49	-7*
Age 17	81	73	-8*

Express 9/100 as a percent.

Answer (9%)**

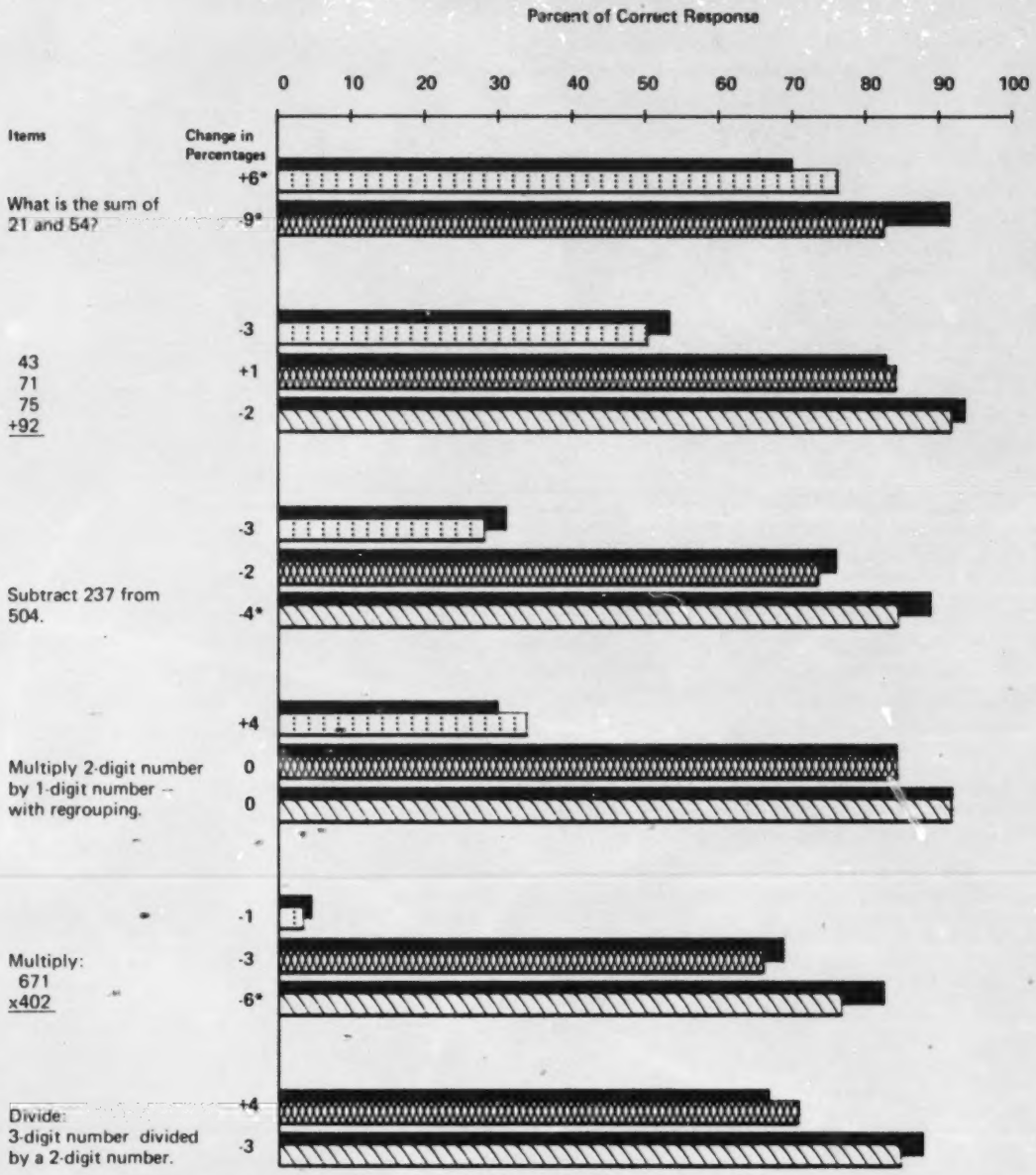
	Percent Correct 1973	1978	Change
Age 13	43	36	-7*
Age 17	61	53	-8*

**Change is significant at the .05 level.*
***9 without a % sign was accepted as a correct response. Very few students responded this way.*

Performance of 17-year-olds also dropped on several items involving calculation with exponents and roots. The percentage correctly adding two single-digit numbers, each raised to the same power, went from 64% in 1973 to 57% in 1978. The percentage who correctly multiplied 2 one-digit numbers, each raised to the same power, dropped 11 percentage points, from 80% to 69%. The percentage who correctly estimated the square root of a given two-digit number fell 9 percentage points, from 52% to 43%.

Sets were an important element of the mathematics curriculum introduced in the 1960s. They have received less emphasis in the curriculum of the 1970s. Assessment items dealt with the concepts of union and intersection of sets. A sample set problem from the assessment appears in Exhibit 3.

FIGURE 1. Percentages of Correct Response on Whole Number Computation Items – 1973 and 1978, Ages 9, 13 and 17



*Indicates the difference is significant at the .05 level.

EXHIBIT 3. Percentages of Correct Responses on Skills Item About Sets Assessed in 1973 and 1978, Ages 13 and 17

Set R = {5, 7, 9, 11}
Set S = {7, 9, 11, 13, 15}

What is the INTERSECTION of set R and set S?
Answer (7, 9, 11)

	Percent Correct		Change
	1973	1978	
Age 13	61	55	-5†
Age 17	74	71	-2†

†Figures do not total because of rounding.

Skills related to the concepts of union and intersection of sets appear to have declined somewhat since 1973, although in a number of cases the declines are not statistically significant (Figure 2). Nine- and 13-year-olds did best on the item in which an intersection was shown pictorially.

Measurement is another basic and essential use of mathematics. Several items for 9-year-olds dealt with making measurements. One item asked 9-year-olds to mark on a clock face a time that is an hour and 10 minutes later than that shown on another clock. Approximately 45% of the 9-year-olds drew the correct time in each assessment. Approximately 35% of the 9-year-olds correctly estimated the length of a given line segment in inches, a decline of 5 percentage points from the first assessment.

Many of the items assessing measurement skills for teenagers concerned conversions between English units. Seventeen-year-olds showed substantial declines on these items; 13-year-olds' performance also tended to drop, although their declines often are not statistically significant (Exhibit 4). It is possible that this skill is receiving less emphasis than previously in the schools, and metric units are receiving more attention.

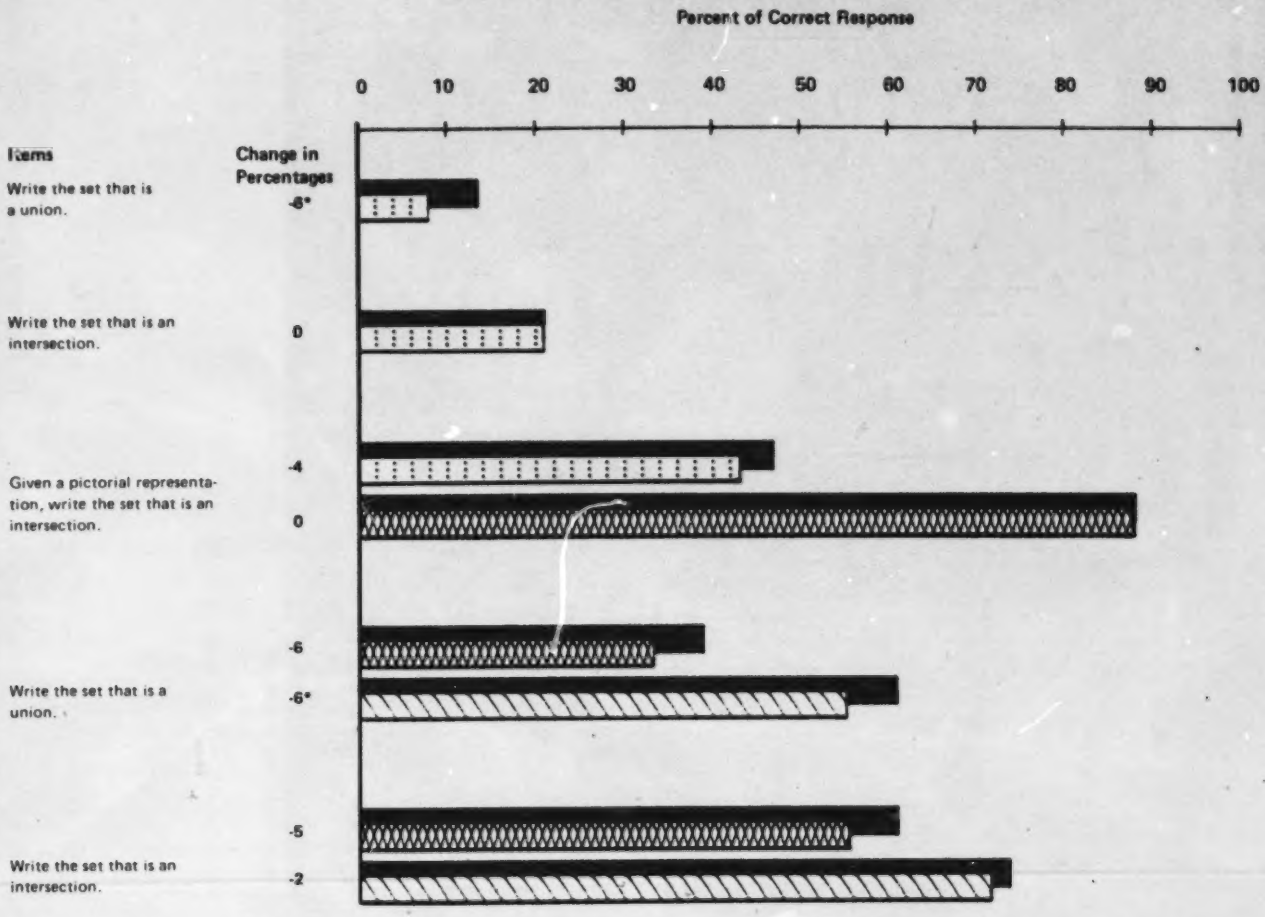
EXHIBIT 4. Percentages of Correct Responses and Changes in Percentages on Measurement Conversion Items Assessed in 1978, Ages 13 and 17

	Age 13		Age 17	
	Percent Correct		Percent Correct	
	1978	Change	1978	Change
x inches = how many feet?	60	- 2	--	--
x pounds = how many ounces?	33	-10*	56	-10*
x gallons = how many pints?	41	- 3	49	-14*
x feet = how many yards?	43	- 3	59	- 8*
Add pounds and ounces	52	- 4	70	- 8*

**Change is significant at the .05 level.*

Quantitative information is often presented in graphs or in tabular form, and it is important that students know how to read information presented in this way. Nearly half the 13-year-olds correctly used a table to find proper amounts of sales tax for various amounts — only a slight decline from the previous assessment. Between 75 and 80% of the 17-year-olds used the table correctly on these problems, dropping about 4 percentage points from 1973.

FIGURE 2. Percentages of Correct Response on Items About Sets – 1973 and 1978, Ages 9, 13 and 17



*Indicates the difference is significant at the .05 level.

Key: Performance in 1973; Performance of 13-Year-Olds – 1978; Performance of 9-Year-Olds – 1978; Performance of 17-Year-Olds – 1978

Thirteen-year-olds improved in their ability to read an air mileage chart — 58% did so successfully in 1978, an increase of 6 percentage points. Seventeen-year-olds' performance did not change appreciably, with about 65% of them using the chart correctly.

Skills in manipulating algebraic symbols include simplifying expressions, solving equations and inequalities, plotting points and graphing equations. Slightly more than one-fifth of the 13-year-olds and just over half of the 17-year-olds in each assessment selected the equivalent expression for a/b = c/d (Exhibit 5). Both 13- and 17-year-olds declined in their ability to multiply 1/3 × a/2. Relatively few 13-year-olds (13% in 1973 dropping to 10% in 1978) correctly completed this task in either year; however, the percentage correct for 17-year-olds dropped rather sharply, from 54 to 39%. Twenty-six percent of the 17-year-olds in 1978 correctly multiplied the binomial factors of a quadratic expression, a decline of 7 percentage points from 1973. Around one-fifth of the 17-year-olds in each assessment successfully gave the binomial factors for a simple quadratic of the form $x^2 + bx + c$.

EXHIBIT 5. Percentages of Correct Responses on Item About Algebraic Expressions Assessed in 1973 and 1978, Ages 13 and 17

If a/b = c/d, then which one of the following statements is TRUE?

- ○ a/d = b/c
- ○ c/b = a/d
- ○ a × c = b × d
- ● a × d = b × c
- ○ I don't know.

	Percent Correct		Change
	1973	1978	
Age 13	24	21	-3
Age 17	53	52	-1

Table 5 displays 17-year-olds' proficiency in solving several equations. Seventeen-year-olds' skills in this area appear to decline.

TABLE 5. Percentages of Correct Responses and Changes in Percentages on Items Involving Equations, Age 17

	Age 17 1978	Change
Find missing numerator in equivalent fraction	72%	-10*
What value of x satisfies $3x + 6 - 14 = x + 2$	34	- 3
Find solution set of $x^2 - 5x + 6 = 0$	18	- 2
Solve for x and y in a system of linear equations	12	- 6*

**Change is significant at the .05 level.*

Plotting points and working with graphs of equations proved difficult for most teenagers. Performance did not change appreciably between assessments. Approximately 20% of the 13-year-olds successfully plotted the point (3,2) on a graph. At age 17, about 20% correctly graphed the equation y = 2x + 1. Approximately 15% of the 17-year-olds determined the slope of the line 2y = 5x - 8, and about 12% gave the y-intercept of this line. Five percent of them determined the equation of a line when the x- and y-intercepts were given, and 3% gave the equation for a circle with the center located at the origin.

Mathematical Understanding

Understanding implies a higher level of cognitive process than recalling facts or using skills. It involves the ability to grasp the principles underlying various knowledges or skills. Exercises measuring understanding often required students to translate knowledge or skills from one form to another, for example, substituting one form of mathematical expression for another or providing an explanation or illustration of a particular knowledge or skill.

Understanding was not heavily emphasized in the first assessment of mathematics, and thus there are relatively few items measuring changes in this area, especially for 9-year-olds. Performance of 17-year-olds declined on these items. The decline for 13-year-olds is very close to significance at the .05 level ($p < .06$) and perhaps should be viewed as a decline (Table 6).

TABLE 6. Changes in Average Performance on Mathematics Understanding Items Assessed in 1973 and 1978, Ages 13 and 17

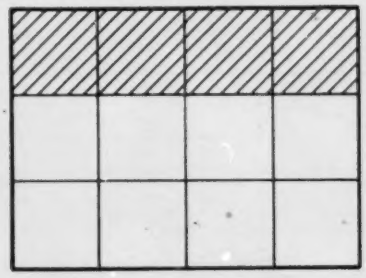
	Number of Items	Average Performance 1973	Average Performance 1978	Change in Average Performance
Age 13	12	52%	50%	-2
Age 17	13	62	58	-4*

**Change is significant at the .05 level.*

The items measuring translation between forms of mathematical expression proved difficult for 9-year-olds. Thirteen-year-olds showed some improvement on these items over the five-year period. Exhibit 6 displays an example item requiring translation from a pictorial to a numeric form. Performance improved substantially between ages 9 and 13. While the percentage of 9-year-olds giving a correct response did not change between assessments, the percentage of 13-year-olds answering correctly increased by 5 percentage points.

EXHIBIT 6. Percentages of Correct Responses on Items Requiring Translation Assessed in 1973 and 1978, Ages 9 and 13

What fractional part of this figure is shaded?

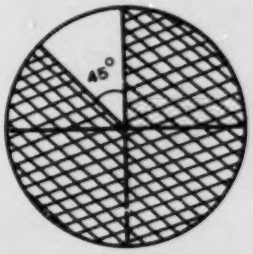


Answer (1/3 or 4/12)

	Percent Correct 1973	Percent Correct 1978	Change
Age 9	19	20	1
Age 13	76	82	5*†

**Change is significant at the .05 level.*
†Figures do not total because of rounding.

Similar results appeared on another item asking for translation of a fraction from a pictorial to a numeric form. Performance of 9-year-olds did not change appreciably on this item, with approximately 11% in each assessment answering correctly, while performance of 13-year-olds improved 8 percentage points, with 71% answering correctly in 1978. About one-fifth of the 13-year-olds in 1978, a decline of 4 percentage points from 1973, determined the fractional part of the circle that is shaded in the following drawing:



The following problem required 13- and 17-year-olds to translate a verbal expression into a symbolic one. About 17% at age 13 and about 46% at age 17 completed the translation successfully in each assessment.

Carol earned D dollars during the week. She spent C dollars for clothes and F dollars for food. Write an expression using D, C and F that shows the number of dollars she had left.

Answer D-(C+F) or D-C-F

Understanding of number theory and the properties of numbers and operations are other topics assessed. Nearly 60% of the 13-year-olds and 75% of the 17-year-olds in each assessment identified a general algebraic expression for an odd number. Approximately 70% of the 9-year-olds in each assess-

ment understood that if a × b = 84, then b × a also equals 84, an example of the commutative property of multiplication. Performance dropped on two items concerning number relationships. As seen in Exhibit 7, fewer 13- and 17-year-olds in 1978 than in 1973 realized that the relationship of the two variables given could not be determined. In a problem dealing with the transitive property of the relation "older than," 50% of the 9-year-olds, 72% of the 13-year-olds and 81% of the 17-year-olds chose the right answer in 1978, declines of 5, 10 and 5 percentage points, respectively. Understanding that the associative property — for example, for addition $(a + b) + c = a + (b + c)$ — applies to addition and multiplication but not to subtraction or division also decreased for the teenaged groups. In 1978, 48% at age 13 and 64% at age 17 demonstrated this understanding, representing declines of 11% for 13-year-olds and 10% for 17-year-olds. Fewer 17-year-olds in 1978 were aware that percent represents a number out of 100. Sixty-eight percent in 1978, 5% fewer than in 1973, identified a given percent as a number out of 100.

EXHIBIT 7. Percentages of Correct Responses on Item About Relationships of Variables Assessed in 1973 and 1978, Ages 13 and 17

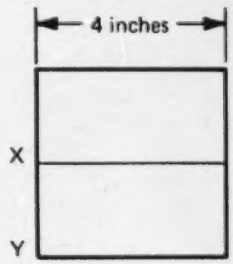
If $a > 5$ and $b > 5$, then

- ○ a equals b.
- ○ a is greater than b.
- ○ b is greater than a.
- ● there is not enough information to determine the relation between a and b.
- ○ I don't know.

	Percent Correct		Change
	1973	1978	
Age 13	67	57	-10*
Age 17	83	76	- 7*

**Change is significant at the .05 level.*

Several items assessed understanding of geometric principles. In both assessments, about two-thirds of the 9-year-olds were aware that if one side of a square is a certain length, an adjoining side has to be the same length. Given the figure following and told that it is a square separated into halves by a line parallel to the base, about a third of the 13-year-olds and just over two-fifths of the 17-year-olds correctly gave the length of line segment XY (2 inches). While performance of 13-year-olds did not change, that of 17-year-olds dropped 9 percentage points. About 30% of the 13-year-olds and 19% of the 17-year-olds in 1978 gave 4 inches as the length of XY, perhaps an indication that they either did not read the question very carefully or did not realize that the term line segment XY does not refer to the entire side of the square.



Approximately half the 13-year-olds in both 1973 and 1978 correctly identified the line segment in a circle that is twice as long as the radius. Many 13-year-olds apparently did not realize that such a line must go through the center of a circle. Performance of 17-year-olds declined on this question, from 77% in 1973 to 73% in 1978.

Seventeen-year-olds responded to several items demanding interpretation of information presented in graphs. Forty-four percent of them successfully determined the period of greatest profit from a graph displaying separate lines for income and expenses, a decline of 7 percentage points from 1973. Performance did not change on two items that asked students to use a graph exhibiting performance of several cars, in which speed is one axis, time the other and the cars' speeds at given times are plotted as curves.

Mathematical Application

Mathematical application involves the use of mathematical knowledge, skills and understanding to solve problems. Problem solving requires judgment — the ability to determine which facts, algorithms or understandings are relevant — as well as the ability to apply the needed processes.

Major emphasis in the application items was on solving typical textbook word or story problems. Some of these problems involve the use of geometry, measurement, probability and statistics, or graphs and tables. A small number of problems were classified as "non-routine," meaning that students most likely would not have encountered similar problems in mathematics textbooks and would have to solve them without benefit of prior experience with similar problems.

All three age groups showed significant average declines on the applications items (Table 7). Nine- and 13-year-olds responded to a relatively small number of items, so results for them should be interpreted with some caution.

TABLE 7. Changes in Average Performance on Mathematics Applications Items Assessed in 1973 and 1978, Ages 9, 13 and 17

	Number of Items	Average Performance 1973	Average Performance 1978	Change in Average Performance
Age 9	9	38%	32%	-6*
Age 13	12	42	38	-3*†
Age 17	25	33	29	-4*

**Change is significant at the .05 level.*
†Figures do not total because of rounding.

Typical word or story problems involving number and numeration concepts were classified, for the purposes of the assessment, as single-step (demanding only one computational operation) or multistep (demanding more than one operation). Many of the multistep problems concerned consumer situations.

Figure 3 displays 9- and 13-year-olds' performances on four single-step word problems. Results for 9-year-olds declined on the easier subtraction task but tended to improve on the more difficult problem, which required subtraction of a three-digit number from a three-digit number. Thirteen-year-olds were considerably more proficient than 9-year-olds on the same subtraction problem; their percentage of success also tended to improve. The numbers in the subtraction word problem given to both 9- and 13-year-olds were also presented as a strictly computational item. Approximately 50% of the 9-year-olds in 1978 subtracted correctly on the computational item, compared with 38% correctly answering the word problem. The difference for 13-year-olds was much smaller, with 85% correctly solving the computational item and 82%, the word problem. As seen in Figure 3, 9-year-olds' abilities on the word problems that required multiplication and division declined rather sharply.

A one-step problem for 17-year-olds required them to divide to find the unit price for kilowatt hours in the electric bill following:

A bill for electricity contains the following information:

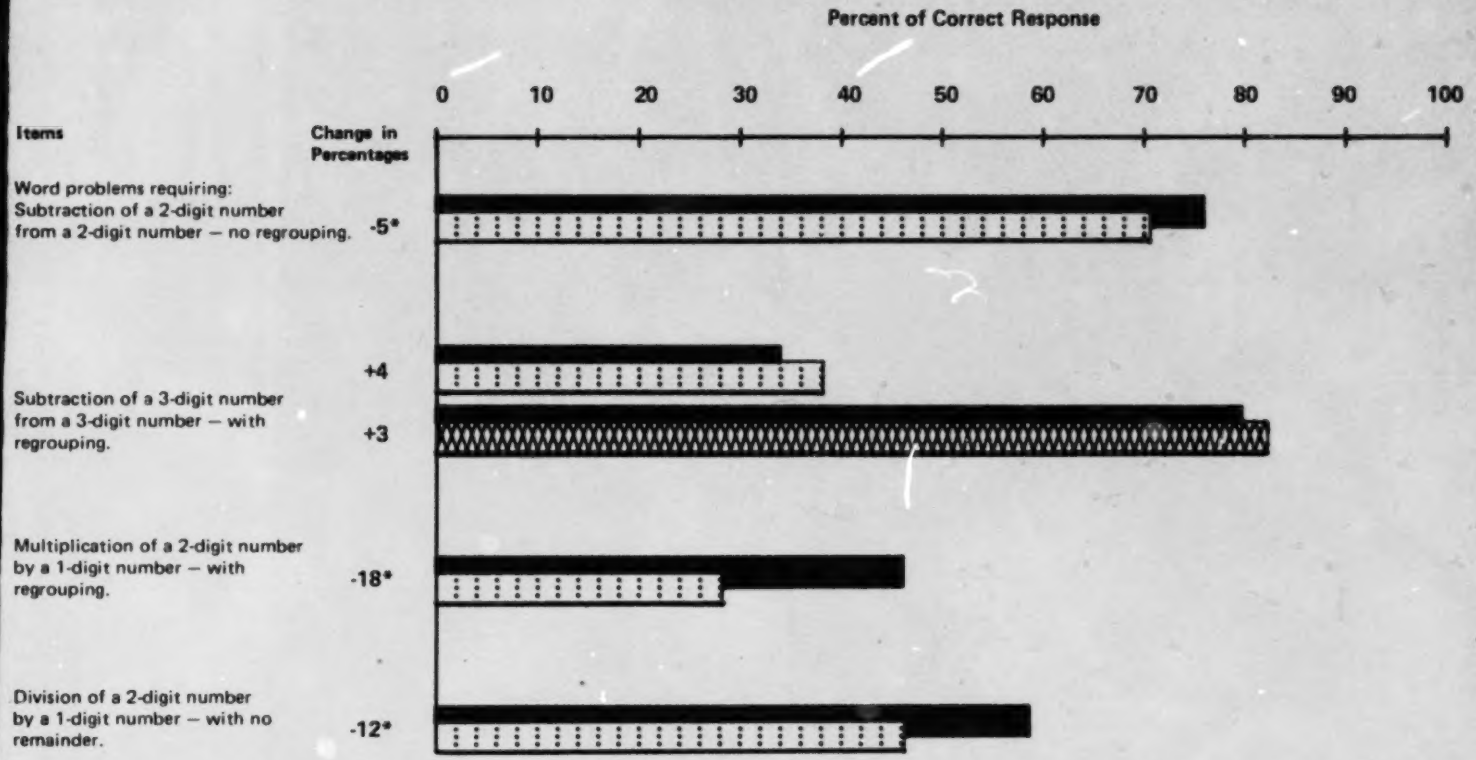
Present Reading	Previous Reading	Consumed	Bill
1548 kw-hr	942 kw-hr	606 kw-hr	$9.09

How much is the customer paying per kilowatt hour for electricity?

Answer (1.5¢)

To find the answer, one would divide $9.09 by 606 kw-hrs. Only 5% of the 17-year-olds answered correctly in 1978, a decline of 7 percentage points from 1973. An additional 4% gave the right numerical answer with no or incorrect units. Approximately 19% in 1978 tried to divide 606 kw-hrs. by $9.09. Another 22% responded "I don't know." About one-third of the incorrect responses were classified as "other unacceptable," meaning that these responses did not fit into any of the cate-

FIGURE 3. Percentages of Correct Response on Single-Step Word Problems — 1973 and 1978, Ages 9 and 13



*Indicates difference is significant at the .05 level.

Key:

Performance in 1973

Performance of 9-Year-Olds — 1978

Performance of 13-Year-Olds — 1978

gories defined in the scoring guide. Some students may have tried to use the extraneous numbers presented in the item.

Nine-, 13- and 17-year-olds were asked to add fractional quantities of liquids pictured in two measuring cups. Students could simply visualize the correct answer from the picture rather than go through the process of adding fractions. About 13% of the 9-year-olds gave the correct answer in each assessment year; the percentage of 13-year-olds responding correctly dropped 13 percentage points, from 69% in 1973 to 56% in 1978; and the percentage of 17-year-olds giving the right answer remained constant, at approximately 83% in each assessment. Percentages of success were considerably lower for an item in which the same fractions pictured in the item discussed above were presented as an addition skill item and were not pictured — 2% of the 9-year-olds, 35% of the 13-year-olds and 67% of the 17-year-olds added correctly on this item. Students typically did better on computational skill exercises than on word problems using the same numbers. In this case, students did much better when they could visualize the solution than when the process of adding fractions was called for.

Seventeen-year-olds responded to a problem that required them to compute one-half of a mixed fraction. Their percentage of success dropped substantially — from 27% to 13% — between assessments.

Several one-step word problems dealt with percent. About one-fifth of the 13-year-olds and just over half of the 17-year-olds solved the following problem:

A hockey team won five of the 20 games it played. What percent of the games did it win?

Answer (25%)

Seventeen-year-olds' performance declined 8 percentage points on this item.

When asked to calculate a certain percent of a given number in a word problem, 15% at age 13 and 40% at age 17 were successful in the second assessment; each age group declined 5 percentage points from its 1973 performance. Percentages of success were similar to those for the previous problem (10% at age 13 and 40% at age 17) on an item asking teenagers to figure the amount of discount on the price of an item when the percent of discount was given. Performance on this item improved by 3 percentage points for 13-year-olds and did not change appreciably for 17-year-olds between assessments. Instead of multiplying, in 1978, about 15% of the 13-year-olds subtracted the percent of discount from the original price, and another 15% divided the price by the percent.

Multistep problems require more than one operation for their solution. Most of these were administered only to 17-year-olds. The problem shown in Exhibit 8 can be solved first by dividing to find the number of five-minute intervals in an hour and then multiplying the number obtained by eight kilometers. Seventeen-year-olds' performance dropped considerably on this item.

EXHIBIT 8. Percentages of Correct Responses on Multistep Problem Assessed in 1973 and 1978, Ages 13 and 17

A car traveled eight kilometers in five minutes. At this speed, how many KILOMETERS could it travel in one hour?

Answer (96 km)

	Percent Correct		Change
	1973	1978	
Age 13	31	28	-3
Age 17	65	56	-10*†

**Change is significant at the .05 level.*
† Figures do not total because of rounding.

Several multistep problems involved the use of percent. Just over one-fourth of the 17-year-olds in each assessment correctly calculated the depreciated value of an object when given the percent of depreciation and the original price. Very few 17-year-olds (about 5% in each assessment) successfully determined the original price of an object when the

sale price of the object and the percent off were given.

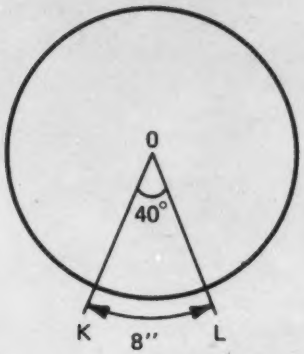
About 10% of the 17-year-olds in both 1973 and 1978 solved a problem concerning a taxi fare, in which a base price and additional increments were given for fractions of a mile traveled and the distance traveled was expressed in decimals.

Seventeen-year-olds had difficulty with several problems requiring the application of geometric concepts. Nearly 20% in both 1973 and 1978 solved a problem involving use of the Pythagorean theorem to find the missing side of a triangle. On another problem, the percentage correctly using the proportionality of similar triangles to find the missing side of a triangle dropped from 10% to 6%.

Nine-year-olds were asked several questions dealing with measurement of time. About one-third of the 9-year-olds in each assessment accurately gave the length of time between two stated times, when they could simply subtract the times and regrouping was not necessary. However, they declined 14 percentage points, from 43% to 29%, in ability to give a time eight hours later than a stated time. This is a more complicated problem since the time period went past 12:00 a.m.; students could not simply add the hours to the original time.

In another measurement problem, 9- and 13-year-olds used a scale drawing and a ruler to determine the distance between two places. About one-quarter of the 9-year-olds and two-thirds of the 13-year-olds gave the correct distance, a decline of 5 percentage points at each age from 1973. Seventeen-year-olds also answered a question about distance represented on a scale drawing. Forty-two percent in 1978 found the correct distance, a decline of 9 percentage points from 1973.

Seventeen-year-olds were asked to determine the circumference of the following circle with center 0:



Twenty percent of them correctly answered 72 inches, a drop of 7 percentage points from 1973. They also were asked to figure the number of cubic feet of concrete needed to fill an area measuring 30 feet x 20 feet x 4 inches. Nine percent were successful in 1978, compared with 13% in the previous assessment. Nearly one-fourth of the 17-year-olds in each assessment simply multiplied all the numbers together, neglecting to convert the numbers to the same units.

Seventeen-year-olds responded to several problems concerning probability and statistics. Success was fairly low on an item involving combinations (5% or less) and on another item involving coin-tossing probabilities (an improvement from 2% to 5%). Exhibit 9 shows two other items — one concerned with probability and the other with combinations — on which performance dropped about 5 percentage points. A drop of 5 percentage points — 18% to 13% — was also seen in 17-year-olds' ability to calculate a weighted average.

EXHIBIT 9. Percentages of Correct Responses on Items Involving Probability and Combinations Assessed in 1973 and 1978, Age 17

2, 3, 4, 4, 5, 6, 8, 8, 9, 10

For a party game each number shown above was painted on a different Ping-Pong ball, and the balls were thoroughly mixed up in a bowl. If a ball is picked from the bowl by a blindfolded person,

what is the probability that the ball will have a 4 on it?

Answer (1/5)

	Percent Correct 1973	1978	Change
Age 17	40	35	-5*

A combination lock on a trunk has three dials, one showing all 26 letters of the alphabet and the other two showing the 10 digits 0 to 9. Assuming that a combination uses a setting on all three dials, how many different combinations are possible?

Answer (2,600)

	Percent Correct 1973	1978	Change
Age 17	20	15	-5*

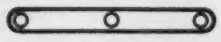
**Change is significant at the .05 level.*

Using information from charts and tables to solve problems is the subject of several other National Assessment items. About one-fourth of the 13-year-olds and one-half of the 17-year-olds found the nutritive values of certain foods listed on a chart and then employed that information in calculating nutritive values of a meal. Both teenaged groups declined on this item, with 13-year-olds dropping 8 percentage points and 17-year-olds dropping 6. About 10% at age 17 in each assessment correctly used a table — showing automobile injuries, pedestrian injuries and total number of injuries for different groups — to determine which group had the highest number of pedestrian injuries per total injuries.

Some problems were judged to be beyond the scope of a "routine" textbook problem. One such problem for 9- and 13-year-olds appears in Exhibit 10. Thirteen-year-olds' success with this problem did not change over the five-year period. However, it appeared largely beyond the problem-solving abilities of 9-year-olds. Another "nonroutine" problem asked 13- and 17-year-olds to determine the smallest number of votes out of a total number of votes cast a candidate must obtain to get the most votes when several candidates are running. Just under one-third of the 13-year-olds and just below two-thirds of the 17-year-olds answered correctly. Percentages of success on this item did not change between assessments for 17-year-olds, but 13-year-olds' performance dropped 8 percentage points.

EXHIBIT 10. Percentages of Correct Responses and Changes in Percentages on a Nonroutine Problem Assessed in 1978, Ages 9 and 13

Mike's building set has

60 long pieces 

60 short pieces and

60 nuts with bolts.

How many of these can he make?

Answer (12)

	Percent Correct 1978	Change
Age 9	3	1
Age 13	24	0

CHAPTER 2

CHANGES IN GROUP RESULTS

In addition to national results, National Assessment provides data on the performance of various groups of students, defined by region of the country lived in, sex, race, type and size of community lived in, level of their parents' education, and their grade in school. Definitions of these groups appear in Appendix A.

Group results are discussed in terms of averages — averages either for all items used to measure change or for all items measuring change in a particular cognitive process level. Results for specific exercises might differ from average patterns.

Group results can be examined in several ways. First, we can consider the performance of a group relative to the nation in each assessment year, thus determining which groups perform above and which perform below national levels. Second, we can examine whether the achievement of each group went up or down over the period between assessments. Third, we can compare the change in a group's performance over time to the change in the national performance level. These indicators should all be considered for an accurate picture of group performance.

It should be remembered that differences in achievement levels cannot be ascribed solely to membership in a particular group being discussed. Research has shown that any number of socioeconomic, school-related and environmental factors are related to achievement, and caution must be taken not to attribute a cause-and-effect relationship between membership in a specific group and performance.

Figures 4, 5 and 6 show average performances at each age level for various population groups in the two assessment years. In these figures, the national mean percentages of success for each assessment are indicated by the two horizontal lines across the graphs. Mean percentages for different groups are indicated as bars — the solid bars representing performance during the 1972—73 school year and the dotted or hatched bars representing performance during the 1977—78 school year. The bars are displayed in seven groups: the regions of the country (NE = Northeast, C = Central, W = West, SE = Southeast); sex (M = male, F = female); race (W = white, H = Hispanic, B = black); level of parental education (NG = not graduated from high school, GH = graduated from high school, PH = post high school); community type (DU = disadvantaged urban, AU = advantaged urban, ER = extreme rural); community size (BC = big cities, F = fringes around big cities, MC = medium cities, SP = smaller places); and grade in school (3 = third grade, 4 = fourth grade, and so forth). Differences in average percentages between the first and second assessments appear at the bottom of each bar; an asterisk indicates that the difference is statistically significant at the .05 level. Because different sets of items were administered at each age level, average percentages for different age groups cannot be compared.

In looking at the graphs for each age group, we can see that changes in group performance between 1973 and 1978 closely paralleled changes seen for the entire nation. There are some exceptions, most notably at age 9. Nine-year-old blacks, although still below national levels, showed an improvement, while their white counterparts suffered a

FIGURE 4. Average Percentages of Correct Response for Selected Reporting Groups on Mathematics Items — 1973 and 1978, Age 9

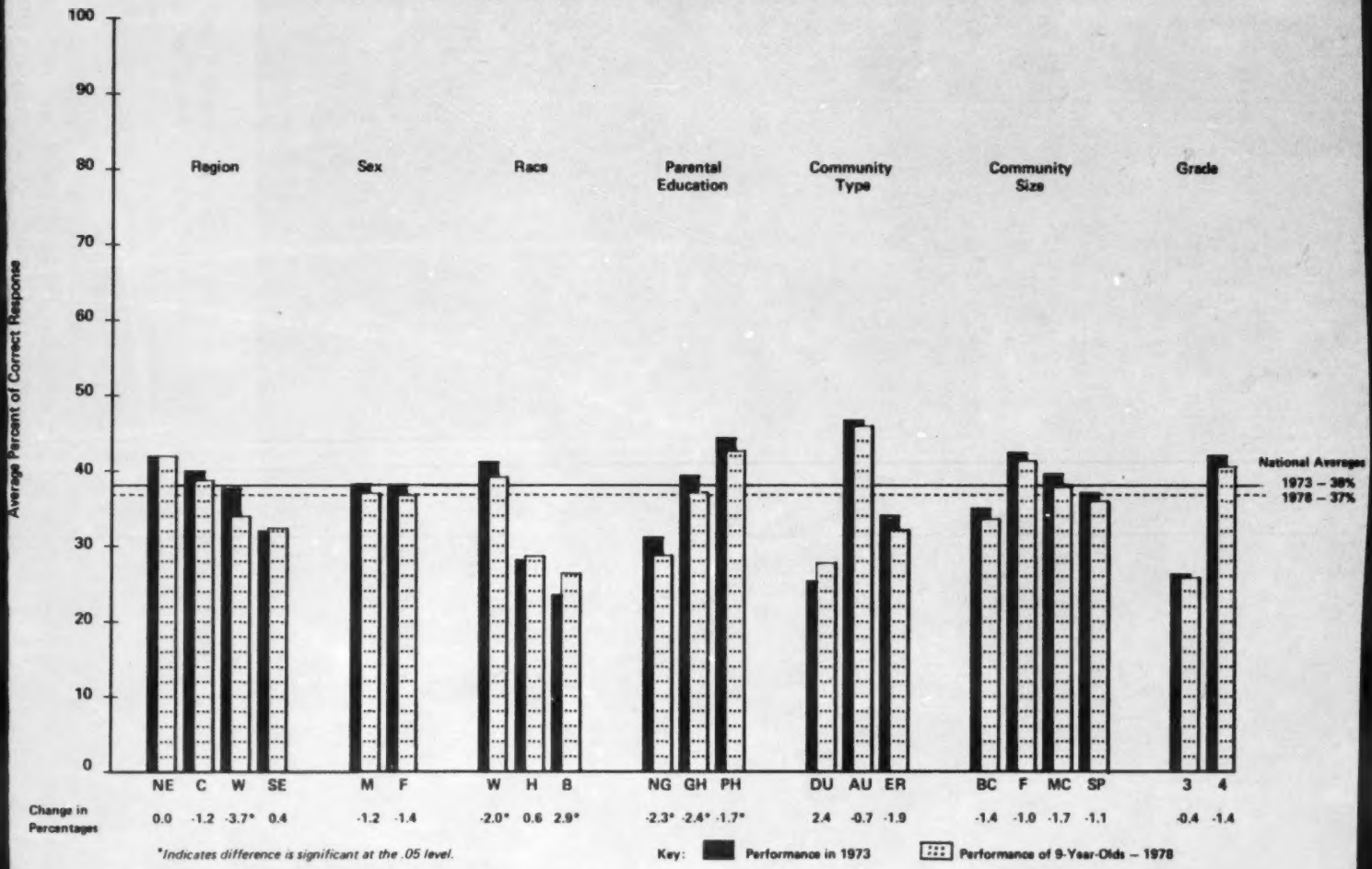


FIGURE 5. Average Percentages of Correct Response for Selected Reporting Groups on Mathematics Items – 1973 and 1978, Age 13

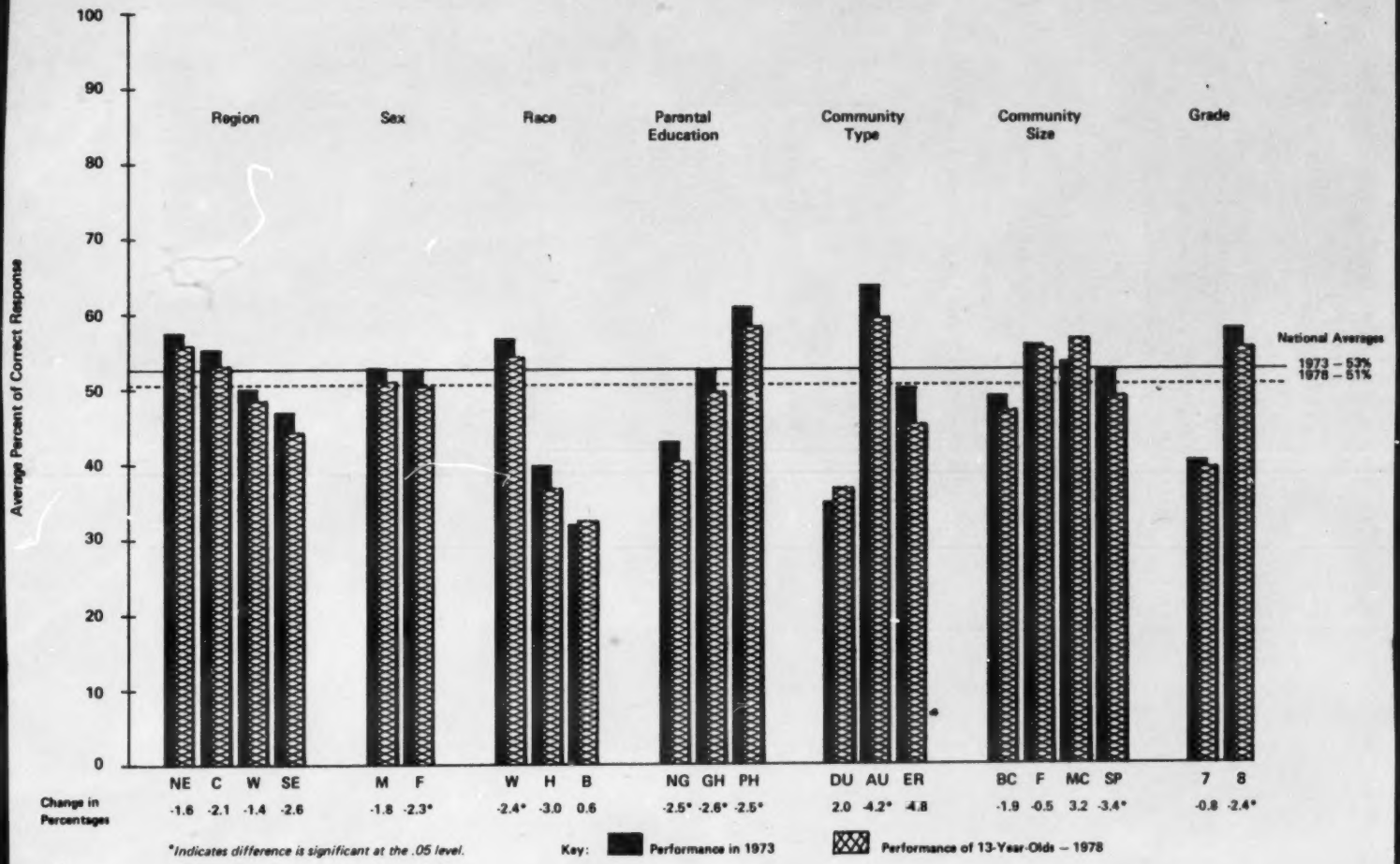
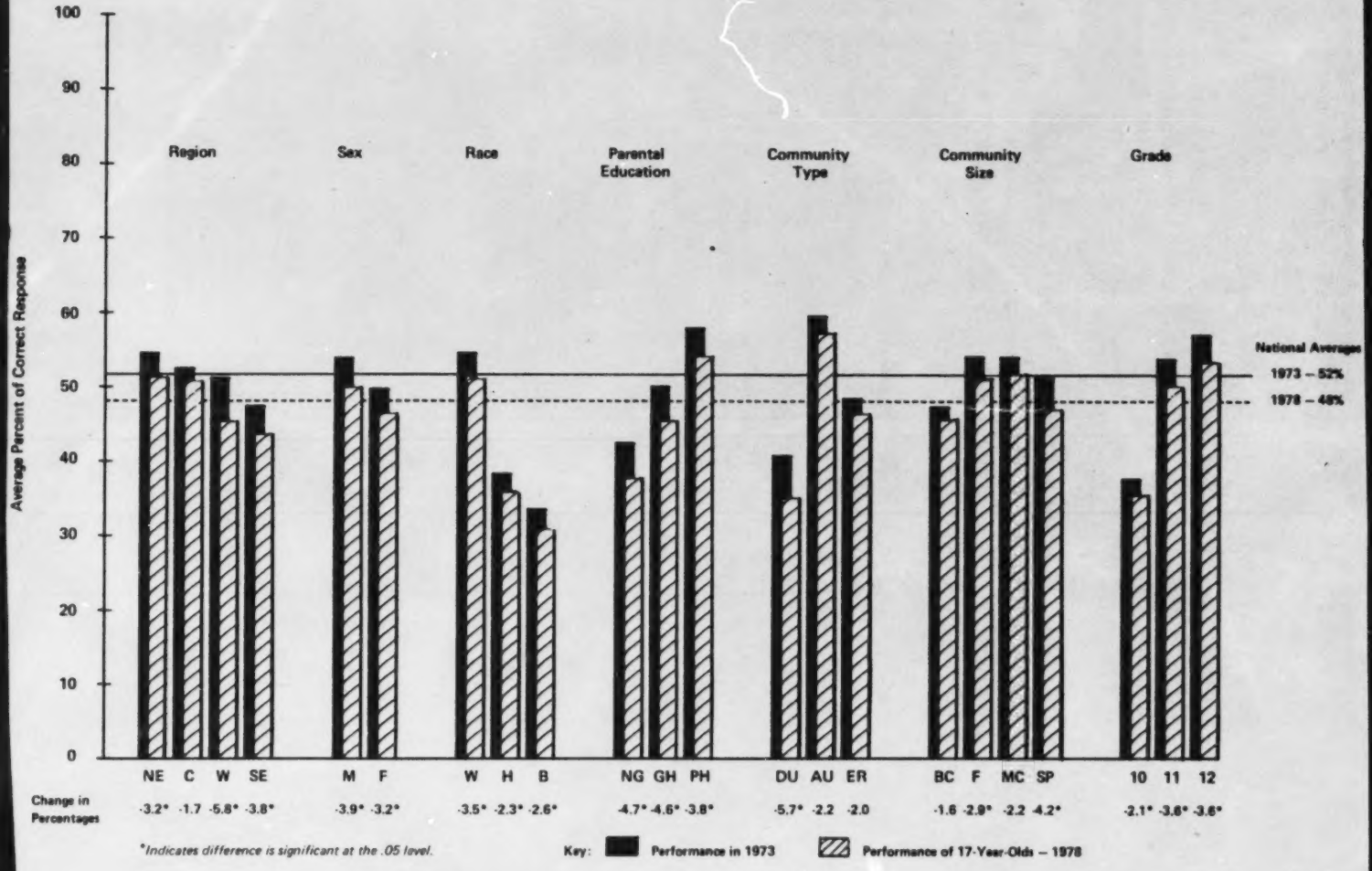


FIGURE 6. Average Percentages of Correct Response for Selected Reporting Groups on Mathematics Items – 1973 and 1978, Age 17



decline. Nine-year-olds living in disadvantaged-urban areas also tended to improve while the national percentage declined slightly, meaning that those in the disadvantaged-urban group at age 9 improved relative to the nation. Those 9-year-olds living in the Southeast, although not showing a significant overall improvement, did improve significantly in the cognitive process areas of skills and understanding and did not decline in the applications area while the other regions did. At age 13, blacks did not show a decline and whites did, although performance of blacks in each assessment remained below that of their white classmates. Achievement of disadvantaged-urban students at this age did not change significantly and tended to improve, although the nation as a whole declined. For 17-year-olds, those living in the Western region tended to decline slightly more than the nation.

The following sections review results for the different groups in more detail. Changes in average performance for the four cognitive process levels — knowledge, skills, understanding and applications — are discussed if they differ from overall average changes in performance.

Region

At all three ages in both 1973 and 1978, students in the Northeastern and Central regions performed above the national level; Western students were either at or below the nation and Southeastern students were below. In general, all regions declined about the same extent as the nation; however, at ages 9 and 17, the West declined slightly more than the nation as a whole.

When results for the different cognitive process levels were considered, National Assessment found that 9-year-olds in the Southeast improved on skills and understanding items, while their age group as a whole did not. They also did not decline on applications items, although the entire group of 9-year-olds did. The Western decline for 9- and 17-year-olds was most noticeable on understanding and applications items.

Sex

At each age level, males and females showed virtually identical declines in performance between 1973 and 1978. Thus, differences between males' and females' achievement remained constant over the five-year period. At ages 9 and 13, there were no differences between overall, average males' and females' performances. However, by age 17, males did better than females in each assessment in terms of overall average achievement.

Results for the different cognitive process levels showed some differences from the overall patterns. In each assessment, 9-year-old males did slightly better than females on applications items; no differences occurred, on the average, for knowledge, skills and understanding items. For 13-year-olds, performance differences between the sexes on the various cognitive process levels were small and generally not consistent across assessments. Seventeen-year-old males did better than their female counterparts on each cognitive process level in both assessments. Differences in performance between 17-year-old males and females were smallest on skills items.

Race

Nine-year-old blacks showed a definite improvement between 1973 and 1978, and their white classmates showed a decline. Hispanos' achievement remained constant (Figure 4). In 1973, an average of 18 percentage points separated black and white 9-year-olds' performances; by 1978, this difference had decreased to 13 percentage points. Results for the different cognitive process levels showed that in knowledge, skills and understanding, performance of blacks at age 9 improved while that of whites either declined or did not change. In the area of applications, blacks'

achievement remained constant (an average of approximately 18% of the black 9-year-olds answered correctly) while whites' percentages declined (a drop from 42% to 35%). The Hispanic group was relatively small; thus, changes in their performance in either direction on the different cognitive process levels are generally not statistically significant.

Thirteen-year-old whites showed an overall decline in performance; 13-year-old blacks did not. Results for Hispanos at age 13 displayed an overall decline similar to that for whites. This same general pattern of changes occurred for all four cognitive levels.

By age 17, favorable trends for blacks were no longer evident. All three racial groups reported upon declined significantly. In both 1973 and 1978, approximately 20 percentage points separated blacks' and whites' performances; approximately 15 percentage points separated whites and Hispanos.

Parental Education

Parental education is related to performance at each of the three age levels: those respondents whose parents had higher levels of education did better, on the average, than those whose parents had lower levels of education. This pattern did not show significant changes during the five-year period spanned by the mathematics assessments and was consistent across all four cognitive process levels.

Type of Community

The three community types analyzed by National Assessment are disadvantaged urban, advantaged urban and extreme rural.

Disadvantaged-urban students at age 9 tended to improve while national performance declined or remained constant — this tendency was most marked on understanding and application items. Advantaged-urban students at this age followed patterns similar to the nation as a whole. Nine-year-old extreme-rural students did not decline as much as the nation on applications items; in other mathematics areas, changes in their performance were similar to those for the nation.

At age 13, disadvantaged-urban students did not show a decline on overall average performance, although the nation as a whole declined. Advantaged-urban and extreme-rural students followed the national pattern.

For 17-year-olds, disadvantaged-urban students showed a larger decline than the other two community-type groups reported upon. When results for the different cognitive process levels are considered, both groups of urban students declined more than the rural group on skills items. Advantaged-urban students showed smaller declines than the other two groups on understanding and applications exercises.

Size of Community

Achievement patterns for the different size-of-community groups did not change between 1973 and 1978. For all three ages, those living in big cities were below the nation and those living in fringes around big cities were above. At age 9, medium-city and smaller-place dwellers performed at the national level. At age 13, those in medium cities tended to perform above national levels. Seventeen-year-olds generally followed the patterns seen for 13-year-olds.

Grade in School

Changes in grade-level performances followed the same pattern as changes in national performance with few exceptions: 13-year-old 7th graders improved relative to the nation on skills items; 17-year-old 10th graders improved relative to the nation on applications items. Those in a higher grade at an age level consistently did better than those in a lower grade in each assessment.

CHAPTER 3

IMPLICATIONS OF THE RESULTS

Many factors must be considered in interpreting changes in mathematics achievement reported by National Assessment. Changes in achievement do not occur in isolation, but take place in the context of societal changes, changes in education practices and changes in curricular emphases. Some changes in performance, although statistically significant, may not be of concern to educators. Changes may reflect deliberate shifts in topics being stressed in the schools or may be temporary fluctuations rather than long-term trends.

To put the changes in mathematics achievement into perspective, National Assessment asked a panel of persons active in mathematics education to comment on the data and consider possible implications. Panel members included: Iris Carl, mathematics resource center specialist and curriculum writer, Houston (Texas) Independent School District, and member of the National Council of Teachers of Mathematics (NCTM) Program and Instructional Affairs Committees; Thomas Carpenter, associate professor of curriculum and instruction, University of Wisconsin, Madison; Robert Dilworth, professor of mathematics, California Institute of Technology, Pasadena; Edgar Edwards Jr., associate director of mathematics (K-12), Virginia Department of Education, Richmond, and member of the NCTM Board of Directors; and James W. Wilson, professor and department head, Mathematics Education Department, University of Georgia, Athens, and member of the NCTM Board of Directors.

This chapter presents selected quotes and a summary of comments made during a two and one-half day meeting at which the panel of consultants informally reviewed and discussed the mathematics results. Observations of the panel members represent their opinions as individuals and do not necessarily reflect the positions of any organizations with which they are associated.

Factors To Be Considered in Interpreting the Data

The panel cautioned that a number of factors must be kept in mind when drawing implications from the data. First, the items used to measure change — those that were included in both the 1973 and 1978 mathematics assessments — are not representative of the entire range of the mathematics curriculum in this country today. The change assessment was limited in size, and results should thus be used only as indicators of changes in the particular areas assessed. The complete 1977—78 mathematics assessment was far more extensive in scope and much more representative of the mathematics curriculum. Readers interested in the current status of young Americans' mathematics achievement should consult NAEP reports describing results of the 1977—78 assessment (*Mathematical Knowledge and Skills*, 1979; *Mathematical Understanding* (forthcoming); *Mathematical Applications*, 1979).

Second, results for specific items as well as changes in average performance must be considered in evaluating the data. While averages are useful in summarizing performance, they tend to mask exceptions and may be unduly affected by a small number of atypical results.

Third, this report describes changes be-

tween two points in time. Additional measures will be needed to determine whether the results presented represent a temporary fluctuation or whether they are indicative of a continuing trend.

Fourth, there may have been shifts over the five-year period, especially at age 17, in the composition of the population being assessed. For example, more 17-year-olds may have been retained in school in 1978 than previously. Shifts in course-taking behavior also may have occurred.

A Historical Perspective

The panel felt that changes in achievement must be considered in light of other changes occurring during the time period spanned by the mathematics assessments. In particular, they focused upon changes in education and, specifically, on changes in mathematics education during the late 1960s and the 1970s.

During the 1960s much federal money was invested in curriculum development in a number of subject areas, partially to help meet the nation's requirements for qualified scientists and technologists. However, it became apparent that the new curricula, among them "new math," did not always meet the needs of all students. Carl noted that the early 1970s ushered in a period of experimentation in teaching approaches, including open classrooms, team teaching, performance contracting, alternative schools and individualized instruction. Introduced with high expectations, many of these approaches failed to provide the hoped-for results.

Public disenchantment with some of the innovations in teaching approaches and concern over achievement declines led to a demand that schools go "back to the basics" and concentrate on the essentials of reading, writing and mathematics. The issue of schools' "accountability" became of increased importance, and "minimal competency" requirements were instituted in many areas to insure that students graduating from high school possessed so-called minimal abilities.

The "back to the basics" movement has been a major force in education during the mid-1970s. In mathematics, back to basics has often implied increased emphasis on computational skills and knowledge of facts and definitions. Textbooks emphasizing basic mathematics were introduced in the early 1970s, but given the time necessary for materials to be widely used in schools, such texts were probably not in widespread use until after the first NAEP mathematics assessment, conducted in 1972—73.

There has been a continued shift in expectations placed on the schools. Schools more and more have been asked to serve as agents of social change — for example, promoting integration, instilling moral values — as well as education institutions. As Wilson noted, "The Osborne study on in-service training showed that teachers ranked non-mathematical content, such as student motivation, learning disabilities and student attitudes toward mathematics, as a topic of higher interest for in-service training than strictly mathematical content topics" (Osborne, 1977, pp. 43—44).

What Do These Changes Mean?

Although the overall results indicate that there has been some decline in mathematics achievement during the 1970s, the panel of consultants cautioned against placing too much emphasis on overall results, and stated that changes on specific types of items or for particular population groups provide much more meaningful information. They also noted that the average changes reported for mathematics are similar to those seen for other areas surveyed by National Assessment during this time period. Factors other than those strictly related to mathematics instruction may be having some impact on performance.

Changes in performance differed by age group, with declines becoming more apparent

for the older students. Nine-year-olds showed very slight changes, with the exception of declines in the application area of problem solving, while 17-year-olds showed measurable drops in each of the cognitive process levels assessed. Dilworth suggested that at age 9, the mathematics curriculum is fairly uniform across the United States. However, by age 17, there is considerably more diversification both in the types of courses offered and course enrollment patterns.

Changes in achievement also differed for different types of items. For example, knowledge of the metric system has received considerable emphasis in the schools during the 1970s in anticipation of the nation's conversion to the metric system. Consultants felt that this emphasis was agreeably reflected in the assessment results, with 13- and 17-year-olds showing substantial gains in their recognition of metric terms. However, the group cautioned that these gains might mean a corresponding lack of attention was being given to other topics. Declines in skill in converting between English units (pints to quarts, feet to inches, and so on) were not seen as cause for particular concern. The panel felt that the level of attention presently given to English units probably is sufficient.

Wilson spoke for the entire group in stating that results for whole number computation, although somewhat ambiguous, were generally satisfactory. Although there were fluctuations in computational abilities over the five-year period, performance was generally high, and declines were, in large part, offset by gains.

The panel was highly concerned with the generally low performance on problem-solving (application) items and with the declines on these items that had occurred since 1973. At each age, fewer students appeared proficient in solving problems deemed appropriate for their age level. For example, 9-year-olds' performance dropped considerably on several one-step word problems. Generally, fewer 13- and 17-year-olds could solve multistep, complex problems or could deal with topics such as percent and probability and statistics.

A number of factors were seen as contributing to these declines. As noted previously, the emphasis on "back to the basics" has often resulted in a narrowing of the curriculum, with more attention focused on computational skills and knowledge of facts and definitions and less time spent on problem solving. As Wilson stated, "Children are given very little opportunity to get into problem-solving activities." Carl concurred: "Back-to-the-basics has stripped youngsters of the chance to practice problem-solving skills."

Textbooks that have come into widespread use during the 1970s have adopted a simplified approach to problem solving. Problems are often presented in a form that uses few words (e.g., 52 marbles. 17 taken away. How many left?). Often, all addition word problems are presented in the addition section of a textbook, all subtraction word problems in the subtraction section, and so forth, so that students do not gain experience in determining which operation is appropriate to the situation presented.

Vocabulary in problems can cause difficulties. As Edwards said, "If students aren't familiar with the vocabulary, they may not be successful in solving problems even though they may have mastered the necessary computational skills." However, the panel felt that potential vocabulary difficulties should not result in the elimination or oversimplification of all word problems in textbooks. Wilson noted that although the inclusion of a wide variety of problems in textbooks may not guarantee that teachers will teach problem solving, it is a necessary first step. If a variety of problems are included in textbooks, there is the potential for teachers to use them as they see fit, and students may at least be exposed to different types of problems.

In conjunction with the simplification of problems in textbooks, the consultants identified the development of a mechanistic approach to problem solving. Teachers often try

to train students to look for clues, or "key words," to decide which operation to use instead of encouraging students to think through the logic of the problem situation posed. Carl noted that there has been an attempt to reduce problem solving to an algorithmic approach, so that students are taught to apply formulas to problem situations. Carpenter felt that "there has been an attempt to remove any kind of thinking. Some texts present whole pages of identical problems, and this does not encourage students to work out the processes needed for themselves."

The increased emphasis on testing brought about by demands for accountability and minimal competency has affected the mathematics curriculum. Increased testing may result in emphasis on areas that are easy to teach and easy to test. As Carpenter stated, "Problem solving is not as amenable to change as other areas. Rote drill and memorization can prepare students for certain types of tests, but problem solving demands more effort, both on the part of the teacher and the student."

Mathematics is a subject in which subsequent steps build upon skills and knowledges learned previously. However, the panel felt that many times students are not introduced to higher-level processes until they have completely mastered the requisite lower-level skills and knowledge. Unfortunately, this may result in a substantial reduction in the time available to devote to problem solving. Carl noted that programs used in the Houston (Texas) schools have resulted in children doing better when exposed to a variety of topics than when drilled to a high level of performance on a few skills before being introduced to different concepts.

Answers to percent problems often indicate that students did not understand the underlying concept involved. Consultants were somewhat dismayed that performance on exercises dealing with percent, not high in 1973, had declined farther by 1978. Carpenter stated that percent, like problem solving, is being treated in a more mechanistic "find the formula and apply it" way in the schools, and students do not appear to understand the principles involved.

Probability and statistics was also regarded as an area warranting further attention. The group agreed that concepts of probability and statistics are very important for the consumer and the general public, since more and more decisions are made on the basis of statistical data. However, these subjects do not receive much emphasis in the schools. Dilworth indicated that these are very important concepts that "are not understood by the general populace, even though people are at times immediately affected by them." Teachers do not seem to regard these topics as important and often do not devote much time to them. The group felt that although declines in probability and statistics were understandable, given current practices, acquaintance with these concepts will become increasingly important in the future.

Declines in algebraic manipulation were also seen as a potential area for concern. Although algebra is not usually a required course for high school graduation, National Assessment data show that in 1977—78 about two-thirds of the 17-year-olds reported they had taken one year of Algebra I. However, the panel felt that results were generally low considering the large number of 17-year-olds reporting that they have taken an algebra course, and felt that reported declines indicate a need for continued, careful monitoring.

Changes for selected population groups indicate that, at least for 17-year-olds, little progress has been made in changing the patterns found in 1973. The same inequities are still in evidence. Whites' achievement is about the same amount above that of blacks in each assessment year, and those from advantaged-urban areas performed about the same percentage higher than those from disadvantaged-urban areas. Males' achievement was about the same amount above females' in each mathematics assessment, although achievement differences between the sexes were fairly small.

However, some progress appears to have taken place at the younger age levels for groups traditionally considered disadvantaged — blacks, those living in disadvantaged-urban areas, in big cities and in the Southeast. Consultants cited the massive infusions of Title I funds to disadvantaged schools as a partial explanation for the changes. Edwards explained that changes have been made in Title I programs during the 1970s: "Title I programs in reading and mathematics are now scheduled in addition to, rather than instead of, regular classroom programs in these subjects. So children in Title I programs now get double doses of mathematics and reading."

Consultants noted the difficulty of interpreting results for population groups since people are members of more than one of the defined groups. For example, the "white" group contains a higher percentage of people who live in advantaged-urban and urban-fringe areas, while the "black" group includes more people living in big-city and disadvantaged-urban areas. Performance of large, heterogeneous groups such as region is also difficult to interpret since no clear pattern of changes is apparent and many factors may come into play.

The group felt that further analyses of finer group breakdowns planned by National Assessment, such as region by race and sex by race, would be useful. Additional analyses on differential course-taking behavior would also be helpful.

Recommendations

In light of the data reported by NAEP, the panel felt that a number of recommendations are in order:

1. An expanded definition of what is "basic" in mathematics is crucial to foster students' ability to cope with different types of mathematical problems. Students must be introduced to exercises involving higher-level as well as lower-level cognitive processes.

2. Textbooks should be modified to include a greater variety of problem-solving tasks. If material is not included in textbooks, teachers may be less likely to introduce it. It would be better to include extra material that might not be used than to reduce the material included to the minimum.

3. Teaching of problem solving should receive more emphasis in the schools. It appears that many teachers feel outside pressure to drill for mastery of skills and not to teach problem solving.

4. Success on tests should not be the only criterion used to measure the effectiveness of mathematics programs. Too often the test then becomes the objective, and areas that require more time to teach or are not as easily subject to modification fall by the wayside.

5. Approaches to problem solving should not be of the type that encourage mechanistic, algorithmic methods. It should be emphasized that the ability to analyze a problem situation is equally as important as the correct solution.

6. Research on effective curricula and teaching approaches is needed to provide a clearer idea of strategies that will improve mathematics achievement.

7. Additional research into causes and potential remedies for disparities in groups' performances is needed.

APPENDIX A

DEFINITIONS OF REPORTING GROUPS

National Assessment examines results for various groups of people within the national population. These groups are defined as follows.

Group Definitions

Sex

Results are reported for males and females.

Race

Results are presented for blacks, whites and Hispanos.

Region

The country has been divided into four regions: Northeast, Southeast, Central and West. States included in each region are shown on the following map.

Level of Parental Education

Three categories of parental-education levels are defined by National Assessment, based on students' reports. These categories comprise those whose parents did not graduate from high school, those who have at least one parent who graduated from high school and those who have at least one parent who has had some post-high-school education.

Type of Community

Communities in this category are defined by an occupational profile of the area served by a school as well as by the size of the community in which the school is located.

Advantaged urban. Students in this group attend schools in or around cities with a population greater than 200,000 where a high proportion of the residents are in professional or managerial positions.

Disadvantaged urban. Students in this group attend schools in or around cities with a population greater than 200,000 where a high proportion of the residents are on welfare or are not regularly employed.

Extreme rural. Students in this group attend schools in areas with a population under 10,000 where most of the residents are farmers or farm workers.

Size of Community

Big cities. Students in this group attend schools within the city limits of cities having a 1970 census population over 200,000.

Fringes around big cities. Students in this group attend schools within metropolitan areas (1970 U.S. Bureau of the Census urbanized areas) served by cities having a population greater than 200,000 but outside the city limits.

Medium cities. Students in this group attend schools in cities having a population between 25,000 and 200,000, not classified in the fringes-around-big-cities category.

Smaller places. Students in this group attend schools in communities having a population less than 25,000, not classified in the fringes-around-big-cities category.

Grade Level

Results are categorized for 9-year-olds in the 3rd or 4th grade, 13-year-olds in the 7th or 8th grade and 17-year-olds in the 10th, 11th or 12th grade.

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